

Inequalities 2

1. Solve $\frac{x^3-3x^2+x+1}{x^3+2x^2+3x+2} \leq 0$

2. Solve $\cos 2\theta > 3 \sin \theta + 2$ for θ , where $-\pi < \theta < \pi$.

3. If $a, b, c \geq 0$, use A.M. $\geq$ G.M., or otherwise, show that

$$\frac{c}{a+b} + \frac{a}{b+c} + \frac{b}{c+a} \geq \frac{3}{2}.$$

4. If $a, b, c \in \mathbf{R}$ and $a + b + c = 2$, show that $a^2 + b^2 + c^2 \geq \frac{4}{3}$.

Find the condition for the equality.

5. Given that a, b, c, d are real numbers and
$$\begin{cases} a + b + c + d = 6 \\ a^2 + b^2 + c^2 + d^2 = 12 \end{cases}$$
 find the maximum value of d .

6. Solve $\left| \frac{4}{x-1} \right| \geq 3 \left(1 - \frac{1}{x} \right) = 3 \left(\frac{x-1}{x} \right)$